The effectiveness of the Wigner functions in understanding optimized coherent state-discrimination receivers

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As E. P. Wigner once remarked, there is an "unreasonable effectiveness of mathematics in the natural sciences". Inspired by this idea, we use the Wigner function formalism to explore the physical meaning of the optimal projective measurement minimizing the error probability when distinguishing M-ary phase-shift-keying (PSK) coherent states in a quantum communication system. We start from our recent results concerning the optimization of state-discrimination receivers for continuous-variable communication systems [**1**], where the optimal measurement corresponds to the so-called pretty good measurement (PGM) [**2**]. After introducing the basic ideas of quantum state discrimination and the main results that define the optimal measurement for PSK coherent states, namely the PGM, we turn our attention to the Wigner representation to visualize and better understand the non-classical and non-Gaussian features of the PGM and its relation with the encoded states, clearly showing the connection between the indistinguishability of the encoded states and the quantum features of the PGM. Finally, we analyze how the nature of the PGM changes as both the number M of the PSK states and their energy increase, revealing how the quantum behavior approaches the "classical" limit without reaching it [**3**].

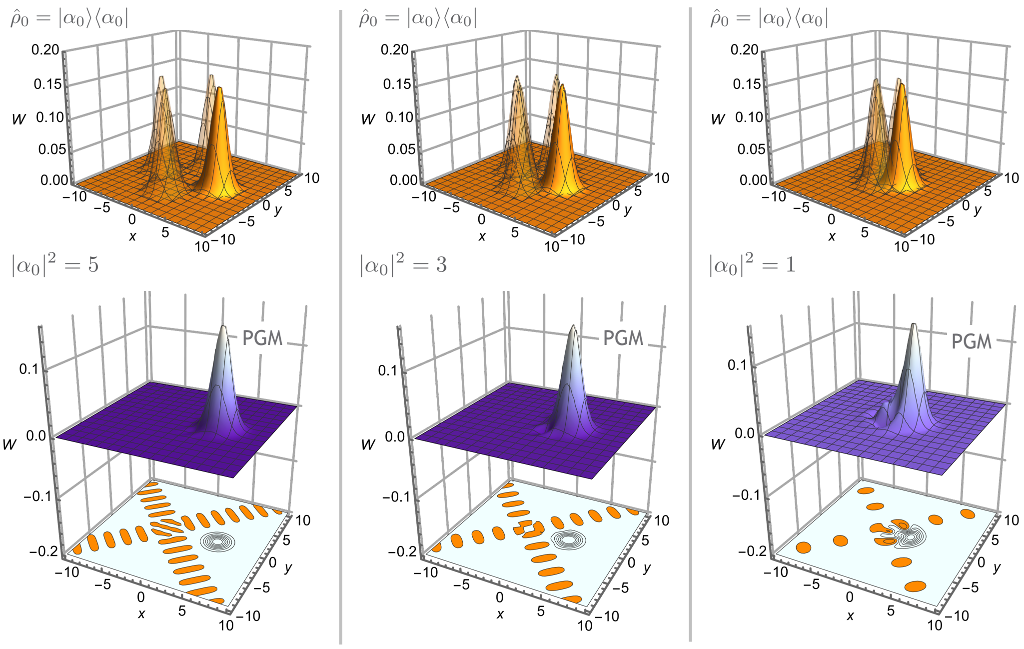

FIG. 1: Plots of the Wigner functions associated with $M = 4$ PSK coherent states for different mean photon values $|\alpha_0|^2$ (top) and the corresponding Wingner function of the PGM (bottom, only the Wigner function associated with the first element of the PGM is plotted, the others are retrieve by applying a suitable phase shift as for the encoded coherent states).

References

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